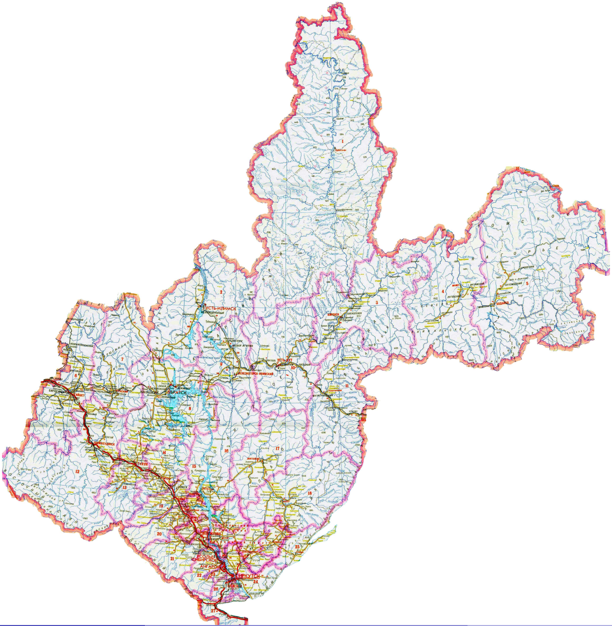
A case study of the regional transportation model

E.A. Nurminski, Shamray N.B.
nurmi@dvo.ru, shamray@dvo.ru

IACP FEB RAS, Vladivostok

VII Moscow Operations Research 2013
Moscow, Russia, October 18, 2013


УСТЬ-ИЛИМСК
УСТЬ-КУТ
ИРКУТСК

General statistics for Irkutsk region

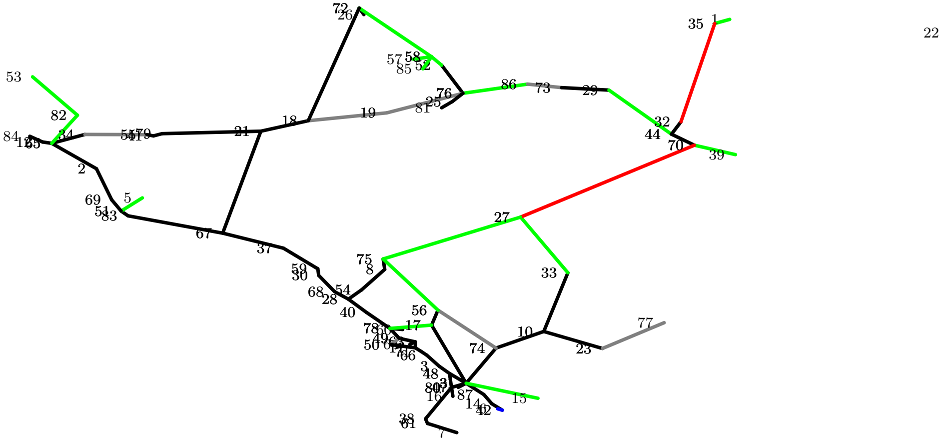
Census 2010:

- Population: 2424355
- Settlements: 86 (1976484)
- Unaccouned: 447871
- Economically active: 61%
- Car ownership: 22.4%

Modal split:

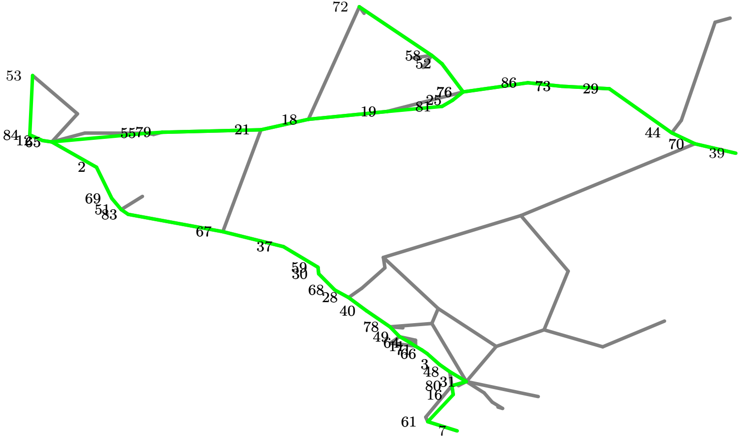
- individual vehicles : 19% ($2204{\cdot}10^3$ trips.est.)
- transit: 65% ($76350{\cdot}10^3$ trips)
- rail: 15% (17143477 trips)
- air: 1% ($2\times$ 865935 trips)

Graph model for automobil road network of Irkutsk district



East Siberian Railway

Graph Model of East Siberian Railway



Methodology — OD-matrix

Modified gravitational model (Voor1955-1958, Carr1956, Wilson1967-1971, Livsh1973, Popkov1983).

$$T_{ij} = A_i O_i B_j D_j f(c_{ij}), \tag{1}$$

where:

T_{ij} — correspondence between i and j;
O_i — outflow from i; D_j — inflow to j;
c_{ij} — unit expenses (time, money, ...) between i and j;
$f(c_{ij})$ — distance function for the OD-pair i, j ;
A_i и B_j — problem parameters.

OD-balance equations:

$$A_i = \left[\sum_{j=1}^{n} B_j f(c_{ij}) \right]^{-1}, \quad B_j = \left[\sum_{i=1}^{m} A_i f(c_{ij}) \right]^{-1}. \tag{2}$$

Balancing

Algorithm due to Fur65,Arrowsmith,Lam81,Schn90:
Initialize:

$$T_{ij}^0 = O_i D_j f(c_{ij}) \left[\sum_{l=1}^{n} D_l f(c_{il}) \right]^{-1},$$

and iterate:

$$\bar{T}_{ij}^k = \begin{cases} T_{ij}^k D_j \left[\sum_{i=1}^{m} T_{ij}^k \right]^{-1}, & \text{if } \sum_{i=1}^{m} T_{ij}^k > D_j, \\ T_{ij}^k & \text{— otherwise} \end{cases}$$

$$Q_i = O_i - \sum_{j=1}^{n} \bar{T}_{ij}^k, \quad R_j = D_j - \sum_{i=1}^{m} \bar{T}_{ij}^k; \tag{3}$$

$$T_{ij}^{k+1} = \bar{T}_{ij}^k + Q_i R_j f(c_{ij}) \left[\sum_{l=1}^{n} R_l f(c_{il}) \right]^{-1}.$$

until input-output balances are satisfied.

Distance functions

Car traffic:

$$f(t_{ij}) = \lambda_{ij} \exp\{-\gamma t_{ij}^{\theta}\},$$

where t_{ij} average time (h) for driving from i to j, γ and θ — scaling coefficients,

Transit, rail and air:

$$f(c_{ij}) = \lambda_{ij} c_{ij}^{-\theta},$$

where c_{ij} — the cost of $i \to j$ trip.

In both cases θ — scaling coefficient, λ_{ij} — attractiveness factor for the $i \to j$ trip.

Attractiveness factor

Borrowed from Статистический ежегодник Транспорт и связь Иркутской области С78 Стат.сб./ Иркутскстат. Иркутск, 2012. 100 с :

$$\lambda_{ij} = \kappa_{ij} \times \tau_j \times E_i^{\alpha_i} \times E_j^{\alpha_j}.$$

where κ_{ij} — the linkage factor for i, j settlements, τ_i, τ_j — tourist attractiveness of i, j, E_i, E_j — income per capita in i, j, α_i, α_j — elasticities.

Значения коэффициентов связности

Тип поселка	Принадлежность	Коэфф. связности		
		Город	**ПГТ**	**Поселок**
Город	Один район	1	1	0.7
	Разные районы	0.4	0.3	0.1
ПГТ	Один район	1	0.7	0.3
	Разные районы	0.3	0.3	0.1
Поселок	Один район	0.7	0.5	0.2
	Разные районы	0.1	0.1	0.1
Село	Один район	0.5	0.2	0.2
	Разные районы	0.1	0.1	0.1

Data preparation

- OD-modeling for an ideal transportation network
- OD-modeling for automobile transportation
  - Individual car transportation
  - Transit
- OD-modeling for rail
- OD-modeling for air

OD for an ideal transportation network

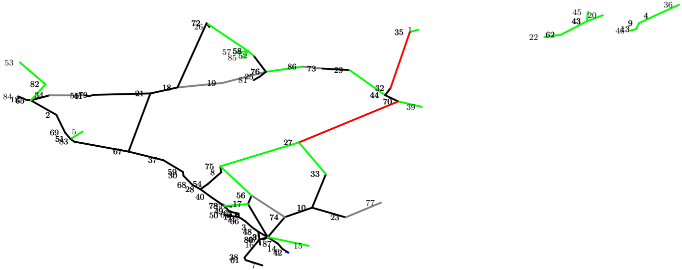
Ideal transportation network corresponds to complete transportation graph.

7482 corresponding pairs, modified gravitational model with the distance function $f(c_{ij}) = \lambda_{ij} c_{ij}^{-\theta}$ (to agree with some popular simulation models). Traffic forcast predicted the following 10 highest flows:

Ангарск	Иркутск	18796.2
Иркутск	Ангарск	3086.11
Иркутск	Братск	1054.48
Братск	Иркутск	2119.95
Шелехов	Иркутск	1262.13
Иркутск	Шелехов	6441.69
Усолье-Сибирское	Иркутск	1038.12
Братск	Усть-Илимск	1601.31
Усть-Илимск	Братск	4074.79

which in general agrees with the observed data on daily labor migration.

Graph model



Graph model for Irkutsk roads

Black – paved road, gray - combined, green — . . . , red — no road, blue — ferries.

Computations were conducted for the backbone net with 5550 OD-pairs without the disconnected part at the left.

OD for the road transportation

- Individual vehicles
- Transit

Individual vehicles

Modified gravitational model with distance function

$$f(t_{ij}) = \lambda_{ij} \exp\{-\gamma t_{ij}^{\theta}\},$$

t_{ij} — minimal time to get from i to j (found by Dijkstra algorithm) with individul times for the edges.

Average velocities for different quality roads were taken as

$v_a = 90$ *км/ч for paved roads;*
$v_a = 80$ *км/ч for combined dress;*
$v_a = 60$ *км/ч for ...;*
$v_a = 40$ *км/ч for no road;*
$v_a = 20$ *км/ч for ferries.*

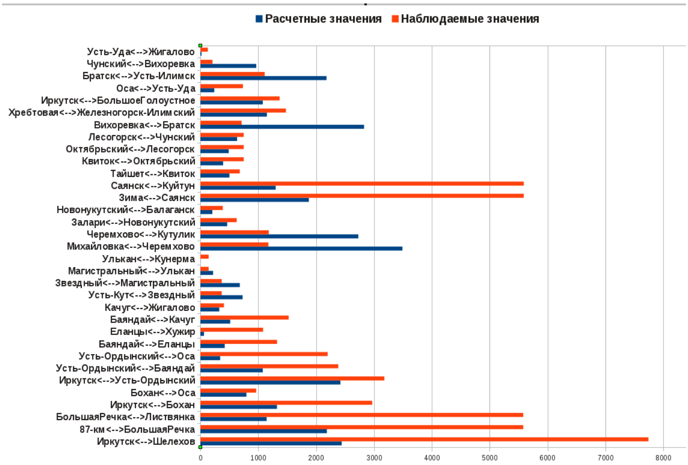
Transit and rail

Gravitational model with the power law for the distance function:

$$f(c_{ij}) = \lambda_{ij} c_{ij}^{-\theta},$$

in all cases c_{ij} was the trip cost, θ is 4 for the transit, and 2 for the rail.

Comparison with observed data



blue — computed traffic, red — observations.

Quality Indicators of Network Topology

- Latora-Marchiori (Latora V., Marchiori M. Efficient behavior of small-world networks // Physical Review Letters, 2001, v. 87)
- Nagurnay-Qiang Nagurney A., Qiang Q. Network Efficiency Measure with Application to Critical Infrastructure Networks // Journal of Global Optimization, 2008, v.40, 261-275.
- Shortest path statistics
- System cost impact indicator (Jenelius E., Petersen T., Mattsson L.G. Road network vulnerability: identifying important links and exposed regions // Transportation Research A, 2006, v. 40, 537-560.)
- Minimal spanning tree (Tero A; Takagi S; Saigusa T; Ito K; Bebber DP; Fricker MD; Yumiki K; Kobayashi R; Nakagaki T. 2010. Rules for biologically inspired adaptive network design. Science. 327(5964): 439-442.)

Notations further on:

- Graph $G = \{V, E\}$;
- V— the set of vertexes with cardinality $n = |V|$,
- $E \subset V \times V$ — the set of edges;
- $W \subset V \times V$ — the set of OD-pairs, $n_W = |W|$.

Latora-Marchiori network efficiency

Network efficiency measure, according to Latora-Marchiori:

$$\epsilon_{RE}(G) = \frac{1}{n(n-1)} \sum_{i,j \in V, i \neq j} d_{ij}^{-1},$$

where d_{ij} — network distance from i to j.
When edges $e \in E$ have lengths $l_e, e \in E$ compute

$$\epsilon_{IE}(G) = \frac{1}{n(n-1)} \sum_{e \in E} l_e^{-1},$$

and Latora-Marchiori coefficient is defined as

$$\epsilon_{LM}(G) = \epsilon_{IE}(G) / \epsilon_{RE}(G).$$

$\epsilon_{LM}(G)$ is maximal (=1) for complete graphs. Irkutsk $\epsilon_{LM} = 0.13894$.

Nagurnay-Qiang

The LM-measure does not take into account the network load. Nagurnay-Qiang suggested the similar measure which takes the load into account.

$$\epsilon_{NQ}(G) = \frac{1}{n_W} \sum_{i,j \in W} q_{ij}^{-1},$$

where $q_{ij} = p_{ij}/d_{ij}$ with p_{ij} —equilibrium cost, d_{ij} — network distance. As it happen ϵ_{NQ} correctly predicted the negative value of the additional road in well-known Braess paradox.

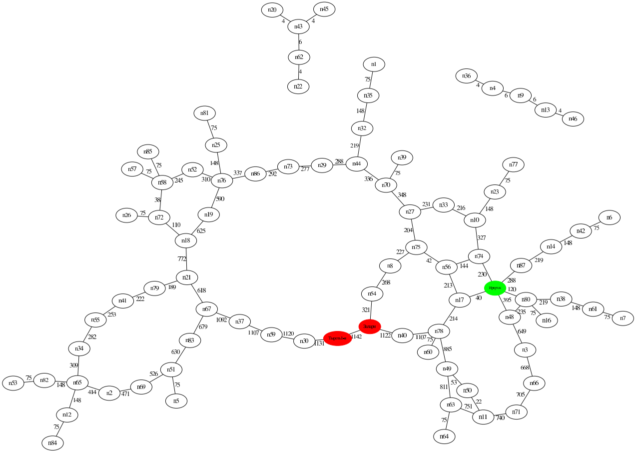
Disadvantage: to estimate $\epsilon_{NQ}(G)$ the difficult equilibrium problem has to be solved.

Shortest paths

"All-or-Nothing"traffic assignment: 5740 routes. Most frequently visited nodes and edges:

Most imp nodes			Most imp edges		
No.	Settl.	Rel imp (%)	Settl.	Settl.	Rel imp (%)
1	Залари	100.0000	Залари	Тыреть1-я	100.0000
2	Тулун	92.4178	Зима	Тыреть1-я	99.0368
3	Черемхово	88.2398	Залари	Кутулик	98.2487
4	Тыреть1-я	87.9304	Зима	Саянск	98.0736
5	Зима	87.0793	Кутулик	Черемхово	96.9352

Overview



The critical edge is red-marked, the green node is Irkutsk.

System cost impact indicator

If removal of the edge e leaves the net connected, SCII is defined as

$$\gamma_G(e) = \frac{1}{n_W(n_W - 1)} \sum_{(i,j)\in W} (p^e_{ij} - p_{ij}),$$

where p_{ij}, p^e_{ij} — are equilibrium marginal costs for the net with and without the edge e.

Modified SCII:

$$\delta_G(e) = \sum_{(i,j)\in W} \alpha_{ij}(p^e_{ij} - p_{ij}),$$

where $\alpha_{ij} = r_{ij}/\sum_{(s,t)\in W} r_{st}$ — is the relative weight of $i \to j$ traffic, makes SCII higher for heavy loaded edges.

Model

Data:

- $Y^0 = \{y_e^0, e \in E\}$ — initial edge capacities, d_w — demand for OD-pair $w \in W$;
- T — a finite set of network accidents, $\mu_{et} \in [0, 1], t \in T, e \in E$, shares of capacity losses;
-

Unknowns:

- $Y = \{y_e, e \in E\}$ — extra capacities
- x_{et}^w – diverted flow from the demand d_w, $w \in D$ on the edge $e \in A$ under the damage scenario t.

Optimization

Objective:

$$\sum_{e \in E} c_e y_e \text{ - total cost of the network expansion}$$

Capacities constraints:

$$\sum_{w \in D} x^w_{et} \leq \mu_{et}(y^0_e + y_e), \quad t \in 0 \bigcup T, e \in E,$$

Flow conservation constraints (Kirchhoff law):

$$\sum_{e \in E^{out}_i} x^w_{et} - \sum_{e \in E^{in}_i} x^w_{et} = \begin{cases} d_w, & i = r, \\ 0, & i \notin w \\ -d_w, & i = s, \quad t \in (0 \bigcup T), w \in D, \end{cases}$$

Computations

LP problem has about

$$N_1 = |E| + |E| \times |D| \times (|T| + 1) + |E| \times (|T| + 1) \text{ variables,}$$

where $|E|$ is a number of edges in transportation network, $|D\|$ is a number of corresponding pairs, $|T|$ is a number of disaster scenarios.

Number of constraints

$$M_1 = (1 + |T|) \times |E| + (1 + |T|) \times |D| \times |V|$$

For $|E| \sim 100, |D| \sim 100, |V| \sim 100, |T| \sim 1000$ the both N_1 and M_1 are of the order of 10^7 and this problem can be referred to as gigabyte optimization problem.

Problem setup

Technology:

$$GMPL \rightarrow NEOS(GUROBI - AMPL)$$

Test problem (failure of 1 of 3 major exit roads out of Irkutsk).

Problem	Failed.	Vars.	Cnstr.	Nonz.	Iter.	Time(sec)
neos-48-56	0	252784	108400	758128	47099	1.74
neos-48-56-1	1	500944	216798	1502718	175646	264.31
neos-48-56-2	2	749104	325196	2247308	267152	935.75
neos-48-56-3	3	997264	433594	2991898	382015	498.68

Acknowledgments

- Государственный контракт № 13-ОК/12 от 12.09.2012 г. на выполнение научно-исследовательской работы по разработке транспортной модели Иркутской области
- РФФИ 13-07-12010 — Облачные и грид-технологии для транспортного моделирования.
- ПреМоЛаб (МФТИ)