Ecomonic equilibrium of traffic flows

Case study of Vladivostok

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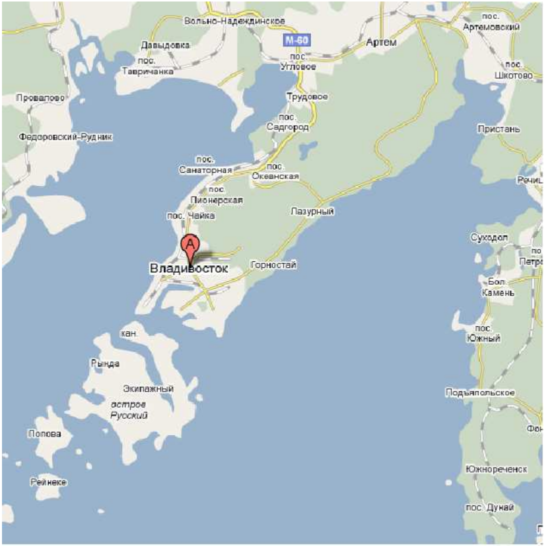
General information

More then 300 mln people within 1000 km zone.

Area of Vladivostok

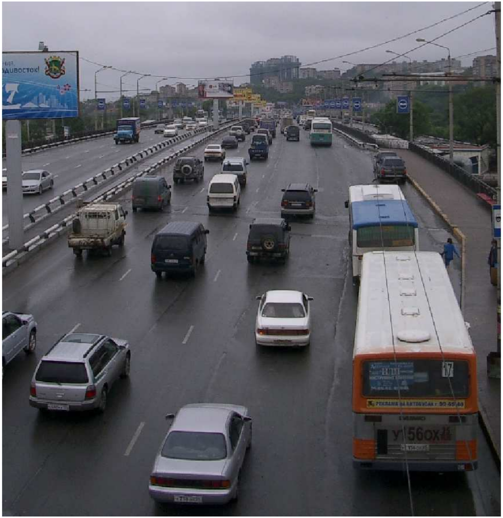


Population 0.55 mln..

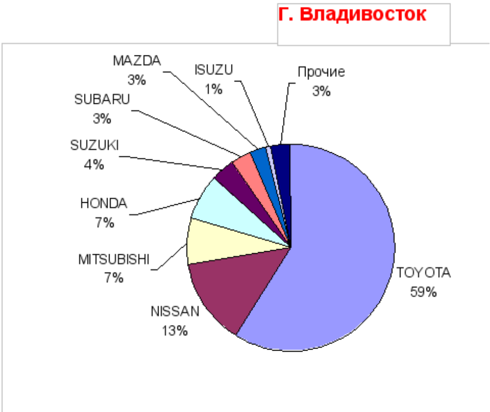
Area of Vladivostok

Population 0.55 mln..

Typical road situation



Car population of Vladivostok



USSR city standard	60 autos/thnd
Vladivostok (2007)	approx. 300 autos/thnd
Russian cities over million	300-350 autos/thnd
European standard	400 autos/thnd

Total number of cars — approx. 180 ths (2007), from those — appox. 155 ths — personal autos.

Detailed graph model of Vladivostok

Graph model statistics

Table: Network characteristics

Leaves (terminal nodes)	1274
Nodes degree 2	241
Nodes degree 3	2521
Nodes degree 4	245
Nodes degree 5	9
Nodes	4290
Nontransit nodes	4049
Arcs	5172
Total length of the streets (km)	1143.37
Average dist between crossings (m)	412.026

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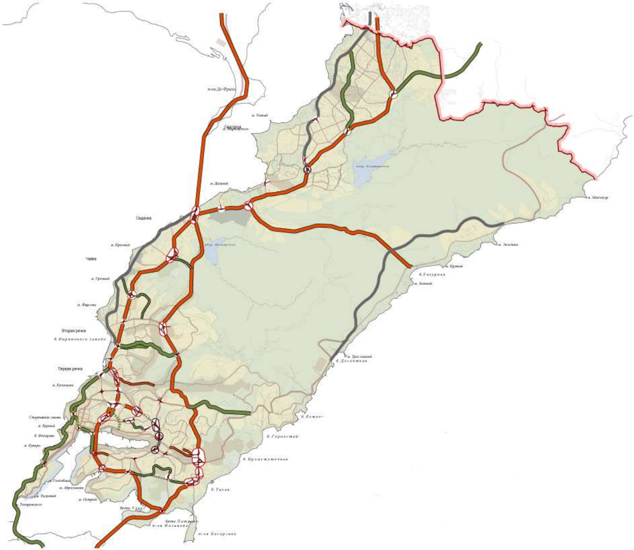
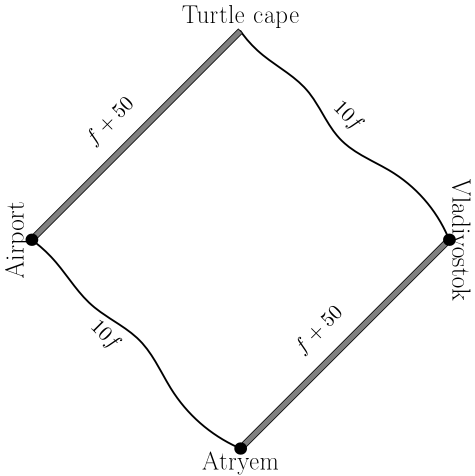
About 6.5 m/car !

Major road building projects



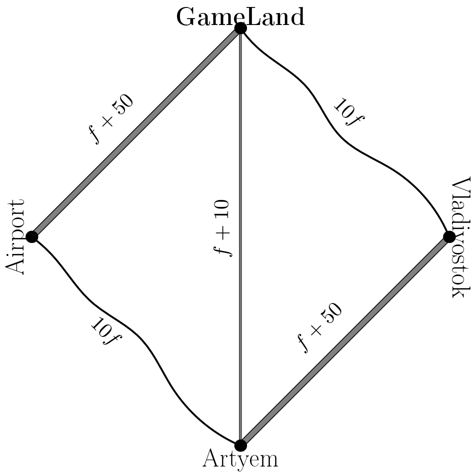
Braess transport paradox — initial stage



Initial state:

- Total demand $A \to B$ — 6 units.
- Traffic splits between 2 routes, user cost — 83.

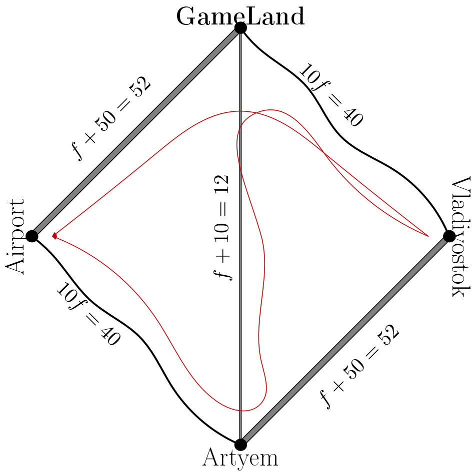
Braess transport paradox — new road added



The road N 3 added:

- Upper route — 2;
- Down route — 2;
- Mixed (down-mid-upper) route — 2.
- User cost — 92. (!)

Braess transport paradox explained



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The route Airport – Artyem – GameLand – Vld at system optimal traffic assignment provides a driver with opportunistic opportunity to save from 83 cost units to 70. Everybody ends up paying 92.

Mathematics behind

Noncooperative equilibrium [1]

No one driver can change his route without increasing his cost.

Mathematically speaking:

Let P_w *— a set of routes, which connect a source-destination pair* $w = (s, d)$ *and* $G_p(x), p \in P_w$ *— marginal costs for these routes as a function of traffic assignment* $x_p, p \in P$ *for all* (P) *routes.*
Then $x_p > 0, p \in P_w$ *implies* $G_p(x) = u_w = \min_{q \in P_w} G_q(x)$.

Back to human language: *Only routes with minimal costs can be used.*

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[1] Wardrope J.G. *Some theoretical aspects of road traffic research*, Proc. of the Inst. of Civil. Eng, Part II, 1952, 1, pp. 325-378.

More mathematics: variational inequalities

Find $x^\bullet \in X$ such that

$$G(x^\bullet)(x - x^\bullet) \geq 0 \quad \text{for any } x \in X,$$

where feasible set

$$X = \{x : \sum_{p \in P_w} x_p = d_w, w \in W, x_p \geq 0\}.$$

set requirements that for a given pair $w = (s, d)$ a prescribed quantity d_w of goods, people, etc should be delivered.

This is a fixed demand problem, there is also elasic demand problem and others.

Complications

- Scale of problems:
  - Chicago Regional Trans Ntwk — approx. 13000 nodes,40000 links, 3 mln od-pairs
  - Southern Ca model — 25000 nodes, 100000 links, etc [2]
- Heavy nonlinearity:
  - Traffic delays are very sensitive to the load: $\tau \sim f^n, n = 4, 5, \ldots$, hence congestion, traffic jams.
  - Nonconvexity — path dependence, unpredictability.
- Stochastic, dynamic and data intensive.

[2] Cited in Nagurnay A, Transport Networks, in:Handbook on Transport Geography

Vladivostok. Golden Horn bridge

Main parameters of the model: 22 nodes, 31 edges, 56 od-pairs.

Numerical results

	bridge	nobridge
Number of constraints	6512	6586
Number of variables	13801	14060
Nonlinear variables	3219	3256
Total cost (carhrs)$\times 10^7$	10.5	6.9
Iterations (MINOS)	3564	3591

Total flow for the bridge: 1400 car/hour.

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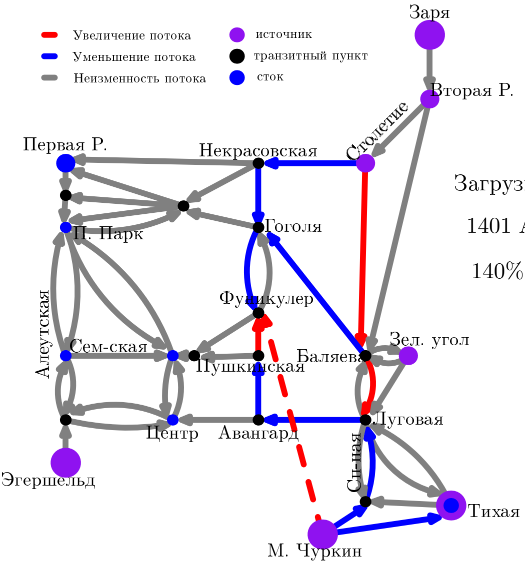


Загрузка моста:

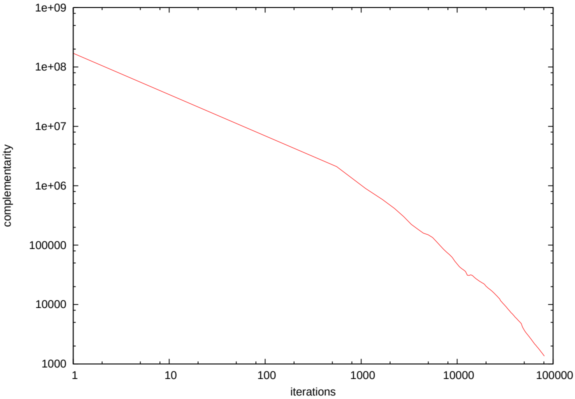
1401 АТС в час на полосу

140% от максимальной

загрузки полосы

Vladivostok test 72x10 case

Complementarity condition



72 od-pairs, 10 routes per pair.