Economic equilibrium of traffic flows
Case study of Vladivostok

E. Nurminski

Far Eastern National University, Vladivostok

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More then 300 mln people within 1000 km zone.
Population 0.55 mln.
Area of Vladivostok

Population 0.55 mln.
Typical road situation
Total number of cars — approx. 180 ths (2007), from those — approx. 155 ths — personal autos.
Detailed graph model of Vladivostok
Graph model statistics

**Table: Network characteristics**

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
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<td>Leaves (terminal nodes)</td>
<td>1274</td>
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<td>241</td>
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<td>Nodes degree 3</td>
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About 6.5 m/car!
Major road building projects
Braess transport paradox — initial stage

Initial state:
- Total demand $A \rightarrow B$ — 6 units.
- Traffic splits between 2 routes, user cost — 83.

Diagram:
- Airport to Turtle Cape
- Turtle Cape to Atryem
- Atryem to Vladivostok
- Vladivostok to Airport

Nodes:
- Airport
- Turtle Cape
- Atryem
- Vladivostok
Braess transport paradox — new road added

The road N 3 added:
- Upper route — 2;
- Down route — 2;
- Mixed (down-mid-upper) route — 2.
- User cost — 92. (!)
The route Airport – Artyem – GameLand – Vld at system optimal traffic assignment provides a driver with opportunistic opportunity to save from 83 cost units to 70. Everybody ends up paying 92.

The road N 3 added:
- Upper route — 2;
- Down route — 2;
- Mixed (down-mid-upper) route — 2.
- User cost — 92. (!)
Mathematics behind

Noncooperative equilibrium \(^1\)

No one driver can change his route without increasing his cost.

Mathematically speaking:

Let \( P_w \) — a set of routes, which connect a source-destination pair \( w = (s, d) \) and \( G_p(x), p \in P_w \) — marginal costs for these routes as a function of traffic assignment \( x_p, p \in P \) for all \( (P) \) routes.

Then \( x_p > 0, p \in P_w \) implies \( G_p(x) = u_w = \min_{q \in P_w} G_q(x) \).

Back to human language: Only routes with minimal costs can be used.

Find \( x^\ast \in X \) such that

\[
G(x^\ast)(x - x^\ast) \geq 0 \quad \text{for any } x \in X,
\]

where feasible set

\[
X = \{x : \sum_{p \in P_w} x_p = d_w, w \in W, x_p \geq 0\}.
\]

set requirements that for a given pair \( w = (s, d) \) a prescribed quantity \( d_w \) of goods, people, etc should be delivered.

This is a fixed demand problem, there is also elastic demand problem and others.
Complications

- Scale of problems:
  - Chicago Regional Trans Ntwk — approx. 13000 nodes, 40000 links, 3 mln od-pairs
  - Southern Ca model — 25000 nodes, 100000 links, etc.

- Heavy nonlinearity:
  - Traffic delays are very sensitive to the load: $\tau \sim f^n$, $n = 4, 5, \ldots$, hence congestion, traffic jams.
  - Nonconvexity — path dependence, unpredictability.

- Stochastic, dynamic and data intensive.

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\(^2\)Cited in Nagurnay A, Transport Networks, in: Handbook on Transport Geography
Numerical results

<table>
<thead>
<tr>
<th></th>
<th>bridge</th>
<th>nobridge</th>
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</thead>
<tbody>
<tr>
<td>Number of constraints</td>
<td>6512</td>
<td>6586</td>
</tr>
<tr>
<td>Number of variables</td>
<td>13801</td>
<td>14060</td>
</tr>
<tr>
<td>Nonlinear variables</td>
<td>3219</td>
<td>3256</td>
</tr>
<tr>
<td>Total cost (carhrs)$\times10^7$</td>
<td>10.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Iterations (MINOS)</td>
<td>3564</td>
<td>3591</td>
</tr>
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</table>

Total flow for the bridge: 1400 car/hour.
Загрузка моста:
1401 АТС в час на полосу
140% от максимальной загрузки полосы
Vladivostok test 72x10 case

Complementarity condition

72 od-pairs, 10 routes per pair.